also used on overland paths when there are enough dips or wiggles in the curves to determine where the reflecting surfaces are located. It will tell the height of the ground and, in many cases, also the tree heights. For example, if a nearby hill shields a midpath, bare hill, or field when the antennas are lowered preventing a direct determination of the elevation, the discontinuity in the height-loss curve caused by reflections from the midpath location can sometimes be used as indicated to calculate the height of the reflection surface.

Limitations of Testing Procedures

It will be apparent from the foregoing that, while microwave testing of proposed transmission paths provides information of great value to the engineer, nevertheless, the testing procedure currently in use does have certain limitations. One of these is the fact that testing is normally limited for reasons of economy to relatively short periods of time, and hence the results do not show the magnitude or frequency of occurrence of variations in atmospheric conditions which may affect transmission. While the tests are usually made only at times when it is believed that transmission is normal, this is not always possible, and the test data must be carefully analyzed in all cases to be sure what conditions are prevailing during the tests. The procedure would be considerably simplified if some simple means of more positive determination of prevailing transmission conditions were available.

Another limitation lies in the fact that the tests are made using unmodulated continuous-wave transmissions, and it is impossible by this means to distinguish between ground reflections along the desired path and reflections from obstacles which may give rise to transmission over some other path of appreciably greater length. This test limitation was recently brought to light in the establishment of an intercity television circuit into a city of moderate size. Path tests using procedures such as those described in this paper indicated that transmission would be satisfactory, although subject to certain reflection effects which were believed to be due to a large body of water in the path. When subsequent tests were made by transmitting a television picture, however, a series of vertical black-and-white bars were observed which were found to be caused by energy overshooting the receiver to strike the face of a large building beyond, from which it was reflected to the face of another large building almost in the transmission path, and from there it was reflected directly into the receiving antenna. This double-reflection path had a length estimated to be some 12,000 feet greater than the direct path, and resulted in a signal delay of approximately 12 microseconds with respect to the main signal. Although the level of this reflected signal was considerably below that of the signal over the direct path, it was nevertheless sufficiently strong to seriously impair transmission, particularly during periods when fading was observed on the direct path. As a result of this experience, consideration is being given to the use of modified test equipment which will employ pulse or modulated-wave transmissions, with receiving equipment which is capable of measuring the amplitude and the time delay of reflected signals.

Conclusions

As the result of extensive observations and tests, it has been concluded that the use of testing techniques similar to those described here are of great assistance to the engineer in selecting sites and determining optimum tower heights for microwave-radio relay stations. Such tests provide information which cannot at present be reliably determined by any other means, and while the testing work is time consuming and involves some expense, the avoidance of serious problems which might otherwise be encountered after permanent construction is completed is believed to be more than adequate justification for use of this procedure on major radio-relay routes.

No Discussion

Computation of Impedance and Efficiency of Transmission Line with High Standing-Wave Ratio

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IT IS ordinarily desirable to operate a radio-frequency or other transmission line with standing-wave ratio as close to unity as possible. When the voltage standing-wave ratio (VSWR) is less than 10, means such as the Smith chart are available for making impedance computations with an accuracy sufficiently good for many purposes. However, in some cases it is desirable or unavoidable to operate the line with high standing-wave ratios. When the VSWR is around 100 the Smith chart is quite satisfactory for reactance but affords no degree of accuracy in reading resistance. This is due to the difficulty of reading even one significant figure on that scale near the periphery, and also to eccentricity in locating the movable arm or the dividers on the center point of the chart.

A simple method is shown for computing the resistance component when the impedance point lies close to the edge of the Smith chart. Attention is also given to precautions to be observed in computing normalized impedance. Similar procedures are shown for admittances. The effect of attenuation is taken into account.

In a related problem, the usual formulas for power and efficiency can lead to considerable error when the VSWR is high. A precise formula is developed herein for the power flowing at a location in a network, in terms involving the magnitude and angle of the reflection coefficient of the load with respect to the source. This is accurate both for lumped-constant networks and for those with distributed

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constants. It is in a form that makes it of particular value in connection with transmission lines in the transverse electromagnetic mode.

On the basis of the power formula, another is readily derived for the efficiency of power transfer along a section of transmission line. The complete formula is more involved than necessary for most applications but can be reduced to various simpler expressions according to the nature of the problem. One of these is the usual formula found for a line with low or medium standing-wave ratio. Another is much more accurate than the latter for cases where the standing-wave ratio is high. It is simple to use, involving only the ohmic resistances and the normalized reactances of the load and input impedances.

Symbols

The following list does not include some symbols that are defined where they occur in the text.

$A_0 = 8.686\alpha h$ = normal or matched attenuation of a length of line in decibels
B_0/G_0 = tangent of the phase angle of Y_0
$g+jb = Y/Y_0$
h = distance along line between two points in units of length
$r+jx = Z/Z_0$
R_0 = real part of the characteristic impedance
S = voltage standing-wave ratio
$Y_0 = 1/Z_0 = G_0(1+jB_0/G_0)$
$Z_0 = R_0(1-jB_0/G_0)$ = characteristic impedance in ohms
α = attenuation constant in nepers per unit length
β = phase constant in radians per unit length
δ = loss angle of dielectric, or power factor when $\delta < 0.1$
η = efficiency
$\theta = \beta h$
$\rho = |\rho|/\underline{2\psi}$ = voltage reflection coefficient
ψ = electrical angle to a point on the line, measured toward the load from a voltage standing-wave maximum

Transformation of Impedance

The first step in solving a particular problem is to decide on the method that is simplest while satisfying the accuracy requirements. A typical case can be set up as follows with reference to Figure 1.

Given: The impedance $Z_1 = R_1 + jX_1$ at one end of the line, and the characteristic resistance R_0. Also α/β, B_0/G_0 and θ or h/λ, which are determined as indicated in the section on attenuation and line length.

Required: To find the impedance $Z_2 = R_2 + jX_2$ at the other end of the line and the efficiency.

General Rules

Determine the simple normalized impedance $(R_1 + jX_1)/R_0$ and set it up on the Smith chart. If the resistance component can be read accurately (say VSWR less than 10 or 20), it is ordinarily satisfactory and simple to use the chart for the entire computation.

If the VSWR is greater than 10 or 20 and the resistance component cannot be read on the Smith chart with the desired accuracy, the following method is recommended.

Procedure

The normalized impedance should properly be computed by use of equation 4 and set up on the Smith chart. However, in most cases the simpler expression $(R_1 + jX_1)/R_0$ is sufficiently accurate for this first step. If working in admittances, use $(G_1 + jB_1)/G_0$. Determine the normalized impedance $r_2 + jx_2$ (or the admittance if required) at the other point of the line by the usual manipulation of the chart. Take account of attenuation by means of equations 9 and 10 and the remarks thereafter. The reactance $X_2 = R_0x_2$ thus found is sufficiently accurate for most purposes. Then subject to the following conditions, the resistance or conductance component can be computed by use of equation 1 and its modifications in equation 2 according to the given and required quantities.

$$R_2 = R_1\frac{1+x_2^2}{1+x_1^2} + R_0(1+x_2^2)\left[\frac{\alpha}{\beta}\theta + \frac{B_0}{G_0}\times\left(\frac{x_2}{1+x_2^2} - \frac{x_1}{1+x_1^2}\right)\right] \quad (1)$$

Observe that R is the ohmic resistance, while x is the normalized reactance. The attenuation of the line in nepers is shown as $(\alpha/\beta)\ \theta$ but can be expressed in other ways, as in equation 10. It is positive when point *2* is on the generator side of point *1*, and negative in the converse case.

Equation 1 is not as tedious to use as its length might indicate, for the terms $(1 + x_1^2)$ and $(1 + x_2^2)$ are each repeated two or three times. Also, in many problems $B_0/G_0 = \alpha/\beta$ affording further simplification.

Conditions

Equations 1 and 2 yield results with accuracy of the order of 1 per cent subject to the following conditions.

1. Neither $1/S_1$ nor $1/S_2$ exceeds 0.17
2. $|B_0/G_0| < 0.1$
3. The normalized impedances r_1+jx_1 and r_2+jx_2 or admittances g_1+jb_1 and g_2+jb_2, whichever are used, must lie in the one-per-cent "permitted region" of Figure 2.
4. The line parameters and given impedance be known to 1-per-cent accuracy. In many problems, this last condition is not satisfied.

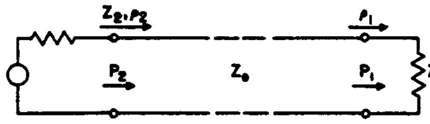

Figure 1. Transmission line of length h

Variations of Formula

Equation 1 gives R_2 when impedance is given. When admittance is given or required, similar formulas can be written with terms shown in the following tabulation:

$$\left\|\begin{array}{lllllll} R_2 & R_1 & x_2^2 & x_1^2 & R_0 & x_2-x_1 & \\ G_2 & G_1 & b_2^2 & b_1^2 & 1/R_0 & -b_2 & b_1 \\ R_2 & G_1R_0^2 & x_2^2 & b_1^2 & R_0 & x_2 & b_1 \\ G_2 & R_1/R_0^2 & b_2^2 & x_1^2 & 1/R_0 & -b_2 & -x_1 \end{array}\right\| \quad (2)$$

The top row shows the terms in equation 1. The second row is for G_2 in terms of G_1, etc. Replace each term in equation 1 with the second row term directly below the original term in the tabulation. Be sure to replace all the x_2^2 by b_2^2 and so on.

It may happen that R_2 is desired, but $r_2 + jx_2$ is not in the permitted region while $g_2 + jb_2$ is in it. Compute G_2 by equation 1 modified by equation 2. Instead of taking the reciprocal of $G_2 + jB_2$, use the equation

$$R_2 = R_0^2G_2|x_2/b_2| \quad (3)$$

where x_2 and b_2 are read on the Smith chart in the usual manner for converting impedances to admittances.

Similarly, when G_2 is to be found and $g_2 + jb_2$ is not permitted, compute R_2 and transpose equation 3 to find G_2. The same means is used to convert between R_1 and G_1 when necessary.

Normalized Impedance and Other Preliminary Considerations

Normalized Impedance

The characteristic impedance and admittance are

$$Z_0 = R_0(1+jX_0/R_0) = R_0(1-jB_0/G_0)$$

$$Y_0 = G_0(1+jB_0/G_0)$$

The impedance and admittance looking toward the load at a point in the line are

$$Z = R+jX$$

$$Y = 1/Z = G+jB$$

Normalized and ohmic impedance are related by

$$r+jx=Z/Z_0=(1/R_0)[R-(B_0/G_0)X+jX] \quad (4)$$

$$R+jX=Z=R_0[r+(B_0/G_0)x+jx] \quad (5)$$

accurate to 1 per cent provided

$$R/|X|=G/|B|<0.1;\ r/|x|=g/|b|<0.1;\ |B_0/G_0|<0.1$$

The equations for admittances are similar, with each impedance term replaced by the corresponding admittance term and the sign before B_0/G_0 reversed.

Caution must be observed to retain the proper sign with X, B, x, or b in the formulas. For example if $X = -300$, it must not be entered in the equation simply as 300. Take special care in this for the terms with coefficient B_0/G_0.

The B_0/G_0 term in these equations can frequently be neglected leaving the customary simple equations for normalized impedance. This is especially so in rough preliminary design computations. The B_0/G_0 terms are most important when the load Q is high, or B_0/G_0 is relatively high such as in spiral lines and resonators and with ordinary lines used at the lower frequencies.

Attenuation and Line Length

If the attenuation of the line is known in decibles per 100 feet, then

$$\frac{\alpha}{\beta}=\frac{0.115(\text{db}/100\ \text{ft})}{30.48}\ \frac{\lambda}{2\pi}\ \text{nepers per radian}$$

Wave length in meters for an ordinary line is

$$\lambda=v/f=300/f\sqrt{\epsilon}$$

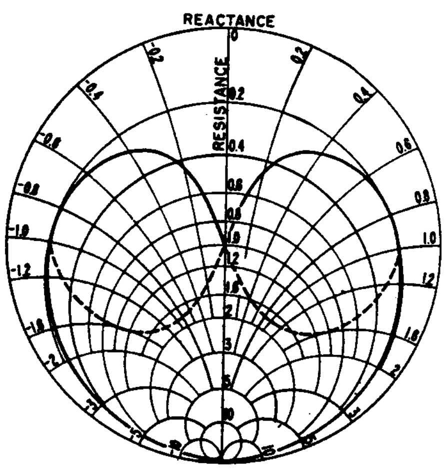

Figure 2. Permitted region for use of formula $S=(1+x^2)/r$. Area outside solid heart-shaped curve drawn on Smith chart is where equation is accurate to within 1 per cent. Area outside dashed curve is where reciprocal of $r+jx$ lies in permitted region, see Appendix I

When attenuation data are not available, compute

$$\alpha/\beta=Rv/2\omega R_0+\delta/2$$

The resistance of a concentric line or a balanced 2-conductor line with copper conductors is

$$R=\sqrt{f}(1/d+1/D)\times10^{-3}\ \text{ohms per foot}$$

where d and D are the diameters in inches of the active surfaces of the conductors and f is in megacycles.

The tangent of the phase angle of the characteristic admittance is

$$B_0/G_0=\alpha/\beta-\delta$$

The line length is

$$h/\lambda=hf\sqrt{\epsilon}/984\ \text{wave lengths}$$

$$\theta=2\pi h/\lambda=hf\sqrt{\epsilon}/156.5\ \text{radians}$$

where h is in feet and f in megacycles per second, while ϵ is the dielectric constant of the medium relative to air.

Derivation of Impedance Transformation Equation

The following equations are well known

$$1/S=(1-|\rho|)/(1+|\rho|)$$

$$|\rho_2|=|\rho_1|\epsilon^{-2\alpha h}=|\rho_1|\epsilon^{-0.230A_0}=|\rho_1|10^{-A_0/10} \quad (6)$$

With respect to a fictitious load with $|\rho|=1.00$

$$|\rho_1|=\epsilon^{-2\alpha h_1}$$

$$|\rho_2|=\epsilon^{-2\alpha h_2}$$

$$h=h_2-h_1$$

It follows that

$$1/S_2=\tanh[0.115A_0+\tanh^{-1}(1/S_1)] \quad (7)$$

Now utilize the results of Appendix I and Figure 2, and note that equation 7 reduces to equation 9 with an accuracy of 1 per cent when neither $1/S_1$ nor $1/S_2$ exceeds 0.17.

$$1/S_1=r_1/(1+x_1^2)=g_1/(1+b_1^2) \quad (8)$$

$$1/S_2=1/S_1+(\alpha/\beta)\theta \quad (9)$$

$$(\alpha/\beta)\theta=\alpha h=0.115A_0 \quad (10)$$

Finally

$$r_2=(1+x_2^2)/S_2$$

$$g_2=(1+b_2^2)/S_2 \quad (11)$$

When equations 8 to 11 are combined with equations 4 and 5 for normalized impedance, equation 1 results. The preceding equations can be used in a step-by-step method of computing impedance, but usually equation 1 provides a shorter computation.

Note the remarks under equation 1 on the sign of the attenuation. When attenuation is greater than 0.01 neper or about 0.1 decibel, it is desirable to take it into account in manipulating the Smith chart. This is done by means of equations 7 and 9, noting that the radial scale "resistance component" (graduated between 0 and 1.0) is identical with $1/S$. The "1-decibel steps" scale on the chart is too coarse for use in many problems.

Power and Efficiency

The net power flowing toward the load at a point on a transmission line Figure 1 is shown in Appendix III to be

$$P={}_fV^2G_0\left(1-|\rho|^2+2|\rho|\frac{B_0}{G_0}\sin 2\psi\right)\ \text{watts} \quad (12)$$

where ${}_fV$ is the rms amplitude of the voltage of the incident wave. The efficiency of power transfer between point *2* on the generator end and point *1* on the load end of a section of line is

$$\eta=\frac{P_1}{P_2}=\frac{1-|\rho_1|^2+2|\rho_1|(B_0/G_0)\sin 2\psi_1}{1-|\rho_2|^2+2|\rho_2|(B_0/G_0)\sin 2\psi_2}\epsilon^{-2\alpha h} \quad (13)$$

This equation can be simplified in many cases according to the conditions of the problem.

Condition A

$$S_1|B_0/G_0|\ll1$$

$$\eta=\frac{1-|\rho_1|^2}{1-|\rho_2|^2}\epsilon^{-2\alpha h}$$

$$=\frac{1/|\rho_1|-|\rho_1|}{1/|\rho_2|-|\rho_2|}=\frac{S_2-1/S_2}{S_1-1/S_1} \quad (14)$$

where $|\rho_2|$ and $|\rho_1|$ are related as in equation 6. The maximum per-cent error, due to the combined effects of numerator and denominator, is roughly

$$\pm100S_1B_0/G_0$$

This is the usual formula found for the efficiency of a radio-frequency transmission line. When the impedance can be computed accurately on the Smith chart, the VSWR being less than 10 or 20, equation 14 should ordinarily be used. The values of $|\rho_1|$ and $|\rho_2|$ can be read on the chart, but it is suggested that one be read and the other computed by equation 6, since they usually differ but little.

Condition B

$$S_2>>1$$

$$\eta=\frac{1/S_1+0.5(B_0/G_0)\sin 2\psi_1}{1/S_2+0.5(B_0/G_0)\sin 2\psi_2} \quad (15)$$

The per-cent error is roughly

$$100/(S_2^2-1)-100/(S_1^2-1)$$

which is

<1 per cent for $S_2>10$ and <10 per cent for $S_2>3.3$

CONDITION C

$S_2>>1$ and $S_1|B_0/G_0|<<1$

$$\eta=S_2/S_1=1/(1+S_1\alpha h)=1/(1+0.115A_0S_1) \quad (16)$$

The error is the algebraic sum of those for conditions A and B. In the expressions involving αh and A_0, this error is increased slightly by an approximation $\tanh x \approx x$ in the use of equation 7.

CONDITION D

$S>>1, |B_0/G_0|<<1$ and $r+jx$ or $g+jb$ in the "permitted region" of Figure 2

$$\eta=\frac{R_1}{R_2}\left(\frac{1+x_2^2}{1+x_1^2}\right)=\frac{G_1}{G_2}\left(\frac{1+b_2^2}{1+b_1^2}\right)$$

$$=\frac{R_1}{R_0^2G_2}\left(\frac{1+b_2^2}{1+x_1^2}\right)=\frac{R_0^2G_1}{R_2}\left(\frac{1+x_2^2}{1+b_1^2}\right) \quad (17)$$

The accuracy of equation 17 is comparable to that of equations 1 and 2 under the conditions listed for those equations.

R_1 and G_1 are the resistive and conductive components of the ohmic impedance and admittance looking toward the load at point *1*, and similarly for R_2 and G_2. On the other hand, x and b are normalized values. The real part of the characteristic impedance is R_0.

In problems where it is desirable to use equations 1 and 2 in computing impedance, equation 17 is generally the simplest one to use for efficiency and the most accurate for numerical computations. The latter equation is identical with the first term on the right-hand side of equation 1 or 2 divided by R_2 or G_2 as the case may be.

The exponential does not appear explicitly in several of the equations 14 to 17. This is not an approximation for small values of attenuation, but the exponential has been absorbed into the body of the equation.

The stated errors for the equations are expressed as a percentage of η, obviously not in addition to η. Furthermore, the impedance and line parameters often will be known inaccurately and can introduce considerable additional error into a numerical computation.

Appendix I. Equations for Boundary of Permitted Region

For 1-per-cent or better accuracy in equations 8 and 11 it is now shown that the impedance point must be in the permitted region of Figure 2 for which

$$|\cot \psi|<0.1S^2/\sqrt{S^2-1} \quad (18)$$

For 10-per-cent accuracy, substitute the coefficient 0.32 for 0.1. When working in admittances, use $\tan \psi$ in place of $\cot \psi$. This gives the same boundary line in Figure 2.

Good formulas for checking rapidly the validity of equations 8 and 11 for 1-per-cent accuracy are

$$r<0.1|x+1/x|$$

$$g<0.1|b+1/b| \quad (19)$$

provided $|x|>0.3$ or $|b|>0.3$ as the case may be. The accuracy condition can be checked visually on the Smith chart, where the circles of constant x must not deviate from the radial direction by more than about 10 degrees at the impedance point in question.

Expression 18 is derived as follows. The complex reflection coefficient can be written

$$\rho=[(S-1)/(S+1)](\cos 2\psi+j\sin 2\psi)$$

The normalized impedance is given by

$$r+jx=(1+\rho)/(1-\rho)=(1+jS\cot\psi)/(S+j\cot\psi)$$

The expression in S and ψ can be demonstrated most readily by substituting the preceding value for ρ and equating as a trigonometrical identity. It is an exact formula. Although it corresponds to an impedance formula with attenuation neglected, the fact that attenuation does not appear does not constitute an approximation. The quantities S and ψ are parameters that completely determine the reflection coefficient at the point where the impedance is $r+jx$.

Equation 8 can be derived from the last formula

$$r/(1+x^2)=(1/S)(S^4+S^2\cot^2\psi)/(S^4+\cot^2\psi)=(1+a)/S$$

The correction term a is always positive.

$$a=(S^2-1)\cot^2\psi/(S^4+\cot^2\psi) \approx [(S^2-1)/S^4]\cot^2\psi \quad (18A)$$

For 1-per-cent error in equation 8, $a=0.01$ and the expression 18 results.

Appendix II. Effect of B_0/G_0 Terms in Impedance Equations

For this discussion, it is convenient to modify equation 1 by the use of the apparent or approximate standing-wave ratio S' and by a result found in Appendix IV.

$$1/S'=R/R_0(1+x^2)\approx G/G_0(1+b^2)$$

$$\sin 2\psi \approx 2x/(1+x^2)\approx -2b/(1+b^2)$$

Then equations 1 and 2 become

$$\frac{1}{S_2'}=\frac{1}{S_1'}+\frac{\alpha}{\beta}\theta+\frac{1}{2}\frac{B_0}{G_0}(\sin 2\psi_2-\sin 2\psi_1) \quad (20)$$

The angle of the reflection coefficient 2ψ can be read on the Smith chart. If working with admittances, read the negative of the angle indicated on the periphery for practical purposes in computing $\sin 2\psi$.

The principal point in question is the effect of the B_0/G_0 terms in the over-all computation and whether they can be neglected. The worst case is when $B_0/G_0=\pm\alpha/\beta$. Then we are interested in the relative value of the quantity

$$(1/2)(\sin 2\psi_2-\sin 2\psi_1) \quad (21)$$

compared to the line length θ. If θ is centered about a voltage maximum or minimum, the quantity 21 is greatest with maximum value of unity. The other extreme is for θ centered midway between these points, when quantity 21 is zero. For small values of θ up to about a radian, the quantity can range between almost doubling the effect of θ down to almost annulling it.

Even with low VSWR, the quantity 21 has the same effect as described on θ. However, when θ is small, it does not have appreciable effect on $1/S$ or the impedance relationship in this case. When θ is large enough so attenuation has appreciable effect on impedances at low VSWR, the quantity 21 is negligible in comparison.

Then the B_0/G_0 terms can be neglected when the standing-wave ratio is low. For higher VSWR, use or omit the B_0/G_0 terms in equations 1, 2, 4, and 20 depending on a mental comparison of the magnitude of quantity 21 in relation to θ in radians.

Appendix III. Derivation of Power Equation

The power flowing at a point in an a-c circuit is given by

$$P=(\text{Real})VI^* \quad (22)$$

This is the real part of the product of the rms complex sinusoid voltage by the conjugate of the corresponding current.

Equation 12 is easily found when the following expressions are substituted in this equation. At a point in a transmission line

$$V={}_fV(1+\rho)$$

$$I={}_fVY_0(1-\rho)$$

$$I^*={}_fV^*Y_0^*(1-\rho^*)$$

$$Y_0^*=G_0(1-jB_0/G_0)$$

$$\rho=(Z-Z_0)/(Z+Z_0)=|\rho|\epsilon^{j2\psi}$$

$$\rho^*=|\rho|\epsilon^{-j2\psi}$$

The power equation 12 can also be demonstrated by a related method. At a point on a line the power flow is given by $P=V^2G$ where V is the rms voltage across the line and G is the conductive component of the admittance looking toward the load. When this is expressed in terms of reflection coefficient, etc., equation 12 results.

The equation can also be demonstrated by integrating the power loss in the series re-

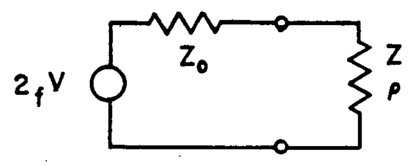

Figure 3. Generator and load

sistance and shunt conductance of the line, between a known load and any other point on the line.

An example and further remarks on the power in a line are given at the end of Appendix V.

Appendix IV. Derivation of Efficiency Equations

The complete equation 13 follows immediately from equation 12. In deriving equation 15 there is used equation 6 and

$$1-|\rho|^2=(1+|\rho|)^2/S=(4|\rho|/S)(1+a') \quad (23)$$

$$a'=1/(S^2-1)$$

The expressions 17 are derived from equation 13 by way of equation 15 with the following steps. From equations 8, 18(A), and 23

$$(1-|\rho|^2)/4|\rho|=(1+a')/S$$
$$=(1+a'-a)r/(1+x^2)$$

$$a'-a\approx(1-\cot^2\psi)/S^2$$

By a deviation similar to that of Appendix I

$$0.5\sin 2\psi=x/(1+x^2)(1-a'')$$

$$a''=[S^2-(S^2-2)\cot^2\psi]/(S^4+\cot^2\psi)$$
$$\approx(1-\cot^2\psi)/S^2$$

This is the same as the approximate equation for $a'-a$ already found. Then

$$1-|\rho|^2+2|\rho|(B_0/G_0)\sin 2\psi$$
$$=(1+a'')4|\rho|[r+(B_0/G_0)x]/(1+x^2)$$
$$=(1+a'')4|\rho|R/R_0(1+x^2)$$

with equation 5 used in the last step. A similar equation is found for admittances. Equation 17 follows readily.

The physical significance of equation 17 is as follows. Subject to the stated conditions, there holds approximately

$$x=\cot\psi$$

$$I=I_{max}\sin\psi$$

where I_{max} is the current standing-wave maximum. The power at any point is

$$P=I^2R$$

where R is the resistive component of the ohmic impedance looking toward the load at the point. Equation 17 follows, since

$$I^2=I_{max}^2\sin^2\psi=I_{max}^2/(1+\cot^2\psi)$$
$$=I_{max}^2/(1+x^2)$$

The value of I_{max} is constant within practical limits for values of S greater than those listed in the statement of errors under the section entitled "Condition D."

Appendix V. Examples and Miscellaneous Remarks

Example of Impedance and Efficiency

A load of $5-j1{,}000$ ohms at 2.0 megacycles is fed by a 60-foot length of *RG-17/U*

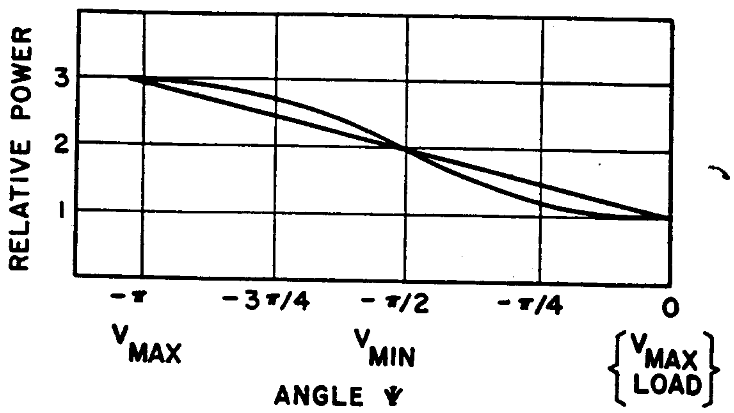

Figure 4. Power flow along line. Straight line is computed power neglecting B_0/G_0 term in equation 12. Wavy line is according to complete equation for case of negligible dielectric loss

cable. What is the input impedance and efficiency?

The attenuation of the cable at 2.0 megacycles is found to be about 0.094 decibel per 100 feet. The loss angle of the polyethylene dielectric is less than 0.0002, and the dielectric constant is 2.26. The given conditions can be summarized as follows, making use of equations in the section on attenuation and line length.

$$R_1+jX_1=5-j1{,}000\text{ohms}$$

$$R_0=52\text{ ohms}$$

$$\alpha/\beta=B_0/G_0=5.6\times10^{-3}$$

$$\theta=1.15\text{radians}(66°)$$

$$h/\lambda=0.183$$

Then $x_1=-1{,}000/52=-19.2$. By the Smith chart $x_2=-0.384$ and so $X_2=-20.0$ ohms. The resistance is found by equation 1 where $1+x_2^2=1.148$ and $1+x_1^2=370$

$$R_2=5\times\frac{1.148}{370}+52\times1.148\times5.6\times10^{-3}\times$$
$$\left(1.15-\frac{0.384}{1.148}+\frac{19.2}{370}\right)$$
$$=(15.5+290)10^{-3}=0.305\text{ ohm}$$

Efficiency, by equation 17 is

$$\eta=15.5\times10^{-3}/0.305=0.051 \quad (5.1\text{ per cent})$$

Reflection Coefficient Greater than Unity

Suppose a loss line with characteristic impedance $Z_0=R_0(1-j0.1)$ is terminated by a lossless load with impedance $Z=jR_0$. Then the normalized load impedance is found to be

$$Z/Z_0=-0.1+j1.0$$

The negative normalized resistance can be visualized if a phasor diagram of the impedances is drawn. It has no implication of negative ohmic resistance.

The reflection coefficient, see Appendix III, is

$$\rho=1.11\underline{/90.3°}$$

Now it is found that there is interchange of power between the incident and reflected waves, but the sum of all the power components is zero at the load.

$$(\text{Real})_f V_f I^*=1.00$$

$$(\text{Real})_r V_r I^*=-1.22$$

$$(\text{Real})_f V_r I^*=0.11$$

$$(\text{Real})_r V_f I^*=0.11$$

The popular conception that the magnitude of the reflection coefficient cannot exceed unity is correct when either the source or load impedance is a pure resistance. With a transmission line, this is usually justified. In fact, when Z_0 includes an appreciable reactive component, it follows inherently that the attenuation per radian is large. Then even if the reflected coefficient exceeds unity at the load, its magnitude is reduced rapidly along the line and soon is less than unity.

Power in Lumped-Element Network

It may be observed that the derivation in Appendix III of equation 12 shows it to be equally valid for either lumped or distributed networks. For the lumped case shown in Figure 3, Z_0 is the generator impedance, Z the load, the $_fV$ is half the open-circuit generator voltage. This equation is not required in most lumped-element network applications since other equations are usually simpler to apply.

With lumped-element networks, the reflection coefficient can be very large. Take for example a load $Z=R+jX$ connected to a generator impedance $Z_0=Z^*=R-jX$. The reflection coefficient is $\rho=jX/R=\pm jQ$. If the open-circuit generator voltage is E, then the power into the load is $P=E^2/4R$ by inspection. It is interesting to compute the power by equation 12 which yields the same result.

One use for the idea of reflection coefficient with lumped element networks is in connection with the application of the principle of superposition. For instance, the load impedance may be replaced by one equal to the source impedance in series with a voltage equal to the actual open-circuit voltage of the source multiplied by the complex reflection coefficient.

Example of Power Flow Along Line

A practical interpretation is given of the power equation 12, showing the net power flowing toward the load at various points along a line with high attenuation.

Let $_fV_a$ and ρ_a be the values of incident rms voltage and reflection coefficient at a voltage maximum. Then at any other point

$$ {}_fV = {}_fV_a \epsilon^{-\psi\alpha/\beta} $$

$$ |\rho| = |\rho_3| \epsilon^{2\psi\alpha/\beta} $$

Now equation 12 can be written

$$ P = {}_fV_a^2 G_0 [\epsilon^{-2\psi\alpha/\beta} - |\rho_a|^2 \epsilon^{2\psi\alpha/\beta} + 2|\rho_a|(B_0/G_0) \sin 2\psi] $$

Suppose $\alpha/\beta = B_0/G_0 = 0.0318$ in which case the dielectric loss is negligible. Let $|\rho_a| = 0.9$ and find the power for various values of ψ between 0 and $-\pi$ radians.

The results are plotted in Figure 4, where arbitrarily $P = 1.00$ at $\psi = 0$, so ${}_fV_a^2 G_0 = 5.25$. As a sample computation, let $\psi = -\pi/4$. Then

$$ -2\psi\alpha/\beta = 0.050 $$

$$ P = 5.25[1.051 - (0.810/1.051) - 2 \times 0.9 \times 0.0318] = 1.18 $$

The curve of power versus position on line in Figure 4 shows that the slope is almost zero at the current minimum and greatest at the current maximum. This agrees with the assumption of negligible dielectric loss, or $B_0/G_0 = \alpha/\beta$. On the other hand, if the conductor loss had been negligible but the dielectric loss appreciable, there would have resulted $B_0/G_0 = -\alpha/\beta$. Then the lobes of the sinusoidal curve of power would have been on the other side of the average power line (that is, where the B_0/G_0 term is neglected). The slope would be greatest at the voltage maximum and almost zero at the voltage minimum which is logical.

The solution of the differential equation of propagation shows uniform voltage and current attenuation of the incident wave and of the reflected wave as they flow in their respective directions. This condition is used in the derivation of equations 12 and 13. However, as shown in the example, this leads to the power dissipation varying nonuniformly along the line. The discrepancy is resolved by the transfer of power between the incident and reflected waves as shown earlier in this appendix.

Reference

1. Reference Data for Radio Engineers. Federal Telephone and Radio Corporation, New York, N. Y., third edition, 1949, chapter 16.

No Discussion

The XY Toll Ticketing System

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IN RECENT years independent telephone companies have become acutely aware of steadily decreasing net profits on short-distance toll-call operation. The recording, for billing purposes, of certain information concerning the toll call has required the assistance of an operator. For this reason, manual toll switching has been retained in spite of the fact that mechanical systems for performing the actual switching functions have been available for many years. Fixed toll charges and rising labor costs have combined to make it extremely difficult for the operating companies to realize a profit on this phase of their business.

One solution to this problem is the granting of free service on these loss-producing toll lines. While this would prevent further losses, resulting increased traffic would create a demand for additional expensive plant equipment and personnel which in turn would produce no revenue.

A second solution is the reduction of the number of people involved in producing the toll billing records known as "toll tickets," by the installation of automatic toll switching equipment incorporating automatic ticketing means. Several attempts have been made to provide such equipment and two basic types have emerged.

One, the verifying type, requires the calling subscriber to dial his own directory number before dialing the number of the desired station. The other, considerably more complex and expensive, is the identifying type wherein the calling station number is automatically found and recorded without effort on the part of the subscriber.[1] The system described herein is of the verifying type.

Any toll ticketing system, manual or automatic, must accomplish the following three things:

1. The information concerning the toll call must be recorded in some manner.
2. The recorded information must be translated and interpreted for use by the ticketing device.
3. The information must finally be presented in a readily usable form.

Recording the Information

Since the primary objective of any toll ticketing system is the reduction of manual labor in producing the toll tickets, the most desirable system would be the one which would reduce the labor factor to zero. In order to achieve such a goal, the designer is confronted with the need for an inexhaustible medium on which to store temporarily the billing information until that information is printed as a toll ticket. Several such storage devices are available, only one of which is economically sound.

The Recorder

The unique magnetic tape recorder shown in Figure 1 forms the heart of the XY toll ticketing system as the intermediate storage device. The tape in the magazine can record information concerning 100 average toll calls before it requires automatic interpreting equipment to print the tickets, simultaneously preparing itself for reuse. One recorder is permanently associated with each toll trunk, operating completely unattended in all of its functions. The recorder records the calling and called subscribers' directory numbers, the duration of the call (elapsed time), the time of day, and the date.

Figure 2 shows the XY switch manufactured by Stromberg-Carlson Company. This is a universal switch for use in any circuit in a step-by-step dial telephone system. Comparison with Figure 1 will show the similarity between the recorder and the switch. Great savings in tooling and manufacturing costs were achieved by using as many identical parts as possible in the two devices. Both are built on the same plate, use two of the same magnets, mount in the same way, and are jacked into the associated trunk relay equipment by means of identical cord and plug assemblies. Thus, the entire design of the XY toll ticketing equipment can be harmonious with standard XY switching equipment.

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